Problem 5 (PS 14)

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a) $U(P,C) = 3P^{0.6}C^{0.3}$

$$|MRS| = \frac{0.6}{0.3}\frac{C}{P} = 2\frac{C}{P}$$

PPF: $\frac{P}{2} + \frac{C}{3} = 10 \Rightarrow 3P + 2C = 60 \Rightarrow |MRT| = \frac{3}{2}$

In optimum: $|MRS| = |MRT| \Rightarrow 2\frac{C}{P} = 3/2 \Rightarrow C = \frac{3}{4}P$

Substituting into PPF: $3P + 2\cdot(\frac{3}{4}P) = 60 \Rightarrow \frac{9}{2}P = 60 \Rightarrow$

$$\Rightarrow \boxed{\begin{array}{l} P = 13\,1/3 \\ C = \frac{3}{4}\cdot P = 10 \end{array}}$$ ← OPTIMUM

$$U(13\,1/3, 10) = 28.32$$

b) Trade $3P = 1C \Rightarrow \frac{P}{C} = 1/3$

Robinson used to have to sacrifice 3 C++ tasks to get 2 Python tasks. Now, he can get 3 Python tasks for 1 C++ task. So it is a good deal for him. New PPF:

$\frac{C}{3} + \frac{P}{9} = 10 \Rightarrow 3C + P = 90$

But, we need to take into account that he needs to pay one python task to be able to trade in the first place. So:

PPF: $3C + P = 89$

$|MRT| = 1/3$ so $|MRS| = |MRT| \Rightarrow 2\frac{C}{P} = 1/3 \Rightarrow$

$C = \frac{1}{6}P$, substituting to PPF:

$3(\frac{1}{6}P) + P = 89 \Rightarrow \boxed{P = \frac{2}{3}\cdot 89 = 59\,1/3}$ ← OPTIMUM

$$3\left(\tfrac{1}{6}P\right) + P = 89 \Rightarrow \boxed{\begin{array}{l} P = \tfrac{2}{3} \cdot 89 = 59\tfrac{1}{3} \\ C = 9\tfrac{8}{9} \end{array}}$$ ← OPTIMUM

$$U\left(59\tfrac{1}{3}, 9\tfrac{8}{9}\right) = 69.13$$

MUCH BIGGER THAN IN a)

Problem 2

child A, a - amount given to child A

child B, b - amount given to child B

money spent on children by parents.

$\boxed{a+b=1000}$

a) Cobb-Douglas

$U(a,b) = \log(a) + \log(b)$

Lagrange multiplier method:

$L(a,b,\lambda) = \log(a) + \log(b) + \lambda(1000-a-b)$

$\frac{\partial L}{\partial a} = \frac{1}{a} - \lambda = 0$

$\lambda = \frac{1}{a}$

$\frac{\partial L}{\partial b} = \frac{1}{b} - \lambda = 0$

$\lambda = \frac{1}{b}$

$\lambda = \lambda$

$\frac{1}{a} = \frac{1}{b}$

$a = b$

$1000 = a + b$

$1000 = a + a$

$1000 = 2a \;/:2$

$a = 500$

$b = 500$

here the kids have the same amount of money.

b) $U(a,b) = \min(a,b)$

here the utility of a parent is determined by the smaller value so it must be $U(a,b) = \min(500, 500) = 500$

because if the utility function was $U(a,b) = \min(400, 600) = 400$, then the utility would be smaller; so the best option is when $a = b = 500$; then the utility function is maximized. so:

$U(a,b) = \min(500, 500) = 500$

$a = b = 500$; each kid have 500.

c) $U(a,b) = a^2 + b^2$

squared values grow faster for larger number; so the best option is to give all money to one child because: $U(a,b) = 1000^2 + 0^2 = 1000000$

if we would divide the money 500 for each the utility is smaller:

$U(a,b) = 500^2 + 500^2 = 250\,000 \cdot 2 = 500\,000$;

$1000000 > 500000$; in this utility function, the best choice for a parent (the highest utility) is when they will give all the money to one of the kids: (two cases)

1) $a = 1000$; $b = 0$

or 2) $a = 0$; $b = 1000$

the final result is the same in each case.

d) $U(a,b) = -\frac{1}{a} - \frac{1}{b}$

Lagrange multiplier method:

$L(a,b,\lambda) = -\frac{1}{a} - \frac{1}{b} + \lambda(1000-a-b)$

$\frac{\partial L}{\partial a} = \frac{1}{a^2} - \lambda = 0$

$\lambda = \frac{1}{a^2}$

$\frac{\partial L}{\partial b} = \frac{1}{b^2} - \lambda = 0$

$\lambda = \frac{1}{b^2}$

$\lambda = \lambda$

$\frac{1}{a^2} = \frac{1}{b^2}$

$a = b$

$2a = 1000$

$a = 500$

$b = 500$

answer

e) $U(a,b) = \max\{a,b\}$ ← this function only considers the larger of the two values (it is the opposite to the $U(a,b) = \min(a,b)$).

So here the answer has two cases: 1) $a = 1000$, $b = 0$ or 2) $a = 0$, $b = 1000$

because: $U(a,b) = \max\{1000, 0\} = 1000$

$U(a,b) = \max\{0, 1000\} = 1000$

(if for example we had $U(a,b) = \max\{400, 600\} = 600$ here the utility is smaller)

the final answer here is:

$\begin{cases} a = 1000 \\ b = 0 \end{cases}$ or $\begin{cases} a = 0 \\ b = 1000 \end{cases}$ they give the same result where the utility function is maximized.

f) $U(a,b) = a^{0,5} + b^{0,5}$

$L(a,b,\lambda) = a^{0.5} + b^{0.5} + \lambda(100-a-b)$

$\frac{\partial L}{\partial a} = \frac{1}{2} a^{-0,5} - \lambda = 0$

$\lambda = \frac{1}{2\sqrt{a}}$

$\frac{\partial L}{\partial b} = \frac{1}{2} b^{-0,5} - \lambda = 0$

$\lambda = \frac{1}{2\sqrt{b}}$

$\lambda = \lambda$

$a = b$

so $2a = 1000$

$a = 500$

$b = 500$

answer

Problem 3

a) $\omega = 24 \qquad S_R + S_J = 24$

$a = \frac{2}{3} \qquad U_R(S_R, S_J) = S_R^{2/3} S_J^{1/3} \qquad U_J(S_J, S_R) = S_J^{2/3} S_R^{1/3}$

$MRS = -\frac{2S_J}{S_R} = -1 \Rightarrow S_R = 2S_J$

$S_R + S_J = 24 \Rightarrow 2S_J + S_J = 24 \Rightarrow S_J = 8$

$S_R = 16$

b) it's the similar situation but reverse one so we get

$S_J = 16 \qquad S_R = 8$

c) To get Pareto optimal allocation
~~Robert~~ increases his consumption by increasing his utility and John decreases his

$MU_R \cdot \Delta S_R + MU_J \cdot \Delta S_J > 0 \qquad \Delta S_R = 1 \qquad \Delta S_J = -1$

$MU_R > MU_J \qquad MU_J < MU_R$

$$\begin{cases} \frac{2}{3} S_R^{-\frac{1}{3}} S_J^{\frac{1}{3}} > \frac{1}{3} S_R^{\frac{2}{3}} S_J^{-\frac{2}{3}} \\ S_J + S_R = 24 \end{cases}$$

$$\begin{cases} S_J > 8 \\ S_R \in (0, 16) \end{cases} \cup \begin{cases} S_R > 8 \\ S_J \in (0, 16) \end{cases}$$

So in Pareto optimal allocation both people will consume more than 8 units

d) $a = \frac{1}{3} \qquad \omega = 24 \qquad S_R + S_J = 24$

$U_R(S_R, S_J) = S_R^{1/3} S_J^{2/3} \qquad U_J(S_J, S_R) = S_J^{1/3} S_R^{2/3}$

$MRS^R_{RJ} = -\frac{S_J}{2S_R} = -1 \Rightarrow 2S_R = S_J$

$S_R + S_J = 2S_R + S_R = 24$

$$\begin{cases} S_R = 8 \\ S_J = 16 \end{cases} \Rightarrow \text{in another case} \begin{cases} S_J = 8 \\ S_R = 16 \end{cases}$$

e) at Pareto Optimal allocation Robert is more happy about John's consumptions so he wants to consume less

and the same with John → he wants to consume less to increase Robert's happiness

So they want to give themselves spaghetti and that's what they will argue about

Wiktoria Nowacka – Homework

Problem set 15

Problem 4

a)

It is complete because the Borda count provides a numerical ranking for each alternative. Given any two alternatives x and y, one will always have a higher or lower count than the other, so a social preference can always be determined.

It is reflexive because any alternative will have a Borda count that is equal to itself, so is at least as good as itself.

It is transitive because if x is more preferable than y, and y is more preferable than z, then x is more preferable than z.

b)

Yes, because if each voter assigns a score to each alternative, then based on this rank x will have smaller score than y, which means that x is socially preferred to y.

c)

Score Voter	1	2
1	x	y
2	z	x
3	y	z

X=1+2=**3**

Y=3+1=**4**

Z=2+3=**5**

X would be the most socially preferable, Y less and Z the least.

d)

Score Voter	1	2
1	x	y
2	y	z
3	z	x

X=1+3=**4**

Y=2+1=**3**

Z=3+2=**5**

Now Y would be the most socially preferable, X less and Z the least.

e)

No it doesn't because when we for example don't consider alternative z in our Borda count from c) and d) then we are not able to choose the best alternative so it doesn't cover with our previous results.

Problem 5

North: $U_N(W_N;W_s) = W_N - W_S^2$

South: $U_S(W_S;W_N) = W_S - W_N^2$

Energy: 4GW

a)

They would allocate it as $W_S=4$, $W_N=0$ and then $U_S=4-0=4$.

b)

They would allocate it as $W_N=4$, $W_S=0$ and then $U_N=4-0=4$.

c)

$W_N=W_S=2$

$U_N(W_N;W_s) = W_N - W_S^2=2-2^2=-2$

$U_S(W_S;W_N) = W_S - W_N^2=2-2^2=-2$

$W_N=W_S=1$

$U_N(W_N;W_s) = W_N - W_S^2=1-1^2=0$

$U_S(W_S;W_N) = W_S - W_N^2=1-1^2=0$

d)

When we know that they have to consume the same amount of energy I will assume that $W_N=W_S=W$.

Total energy consumed will be $2\times W$.

Energy thrown away will be $4-2\times W$.

Then joint utility will be $U=W-W^2$.

To maximize $U \Longrightarrow dU/dW=1-2\times W=0$.

$W=0.5$

Total energy consumed = $2\times 0.5=1$

And energy thrown away will be $4-1=3$.