

Problem 1

Inhabitants of a city consume a private good x (understood as money spent on all private goods a consumer buys, so the price of this good is 1) and a public good g (a public ice-rink). Each of the 1,000 inhabitants of the city has the utility function: $U(x,g) = x - 100/g$, where x is the value of private consumption and g – the size of the public ice-rink in square meters. The cost of the ice-rink is 10 per square meter. Each inhabitant has an income of 1,000.

- Find the Pareto-efficient ice-rink size.
- Assume each inhabitant will bear an equal share in the costs of the ice-rink (i.e., $10g/1000$). The ice-rink size is decided in a poll, where each inhabitant has one vote. What ice-rink size will be chosen in this way?

Problem 2

A public good costs 99 PLN. Three persons take part in a majority vote on whether to provide this good or not (a yes-no vote). Their reservation prices are $r_1 = 90$, $r_2 = 30$, and $r_3 = 30$, respectively. In case of a positive result of the voting, each of these persons bears 1/3 of the total costs of providing the good.

- What will be the result of the voting?
- Now, instead of the majority voting, the persons declare their values of the good, and the Vickrey-Clarke-Groves (VCG) mechanism is introduced to incentivize truth-telling. Who will pay the VCG tax, and in what amount?

Problem 3

Two mines and two laundries use a river. The mines are located upriver, and they use the river to dump untreated sewage. Sewage treatment would generate an additional cost for the mines. Laundries operate downriver, and the river water pollution increases their work cost. Both laundries and mines are lobbying for and against, respectively, a government intervention to reduce river water pollution. They declare the net values as presented in the table, and the Vickrey-Clarke-Groves (VCG) mechanism is introduced. Who and in what amount will pay the VCG tax?

	Government intervention	No government intervention
Mine A		30
Mine B		38
Laundry C	40	
Laundry D	30	

Problem 4

A public good is being considered to be delivered to a society consisting of three individuals: A, B, and C. Individual i , where $i \in \{A, B, C\}$, assigns value v_i to have the good delivered and needs to pay cost c_i for the good provision.

Individual	c_i	v_i
A	100	45
B	100	45
C	100	250

- Suppose the decision about the good provision (or not) will be taken based on a majority yes-no vote, in which each individual will be asked whether she is willing to pay cost c_i for having the good delivered. What will be the result of the vote? Is the outcome Pareto efficient?
- Now suppose individuals declare their values, and this will determine whether the good is provided or not. The Vickrey-Clarke-Groves (VCG) mechanism is introduced. Who and in what amount will pay the tax? Is the outcome Pareto efficient?
- In b, are the individuals incentivized to reveal their values truthfully? Answer using the example of individual A. Assume that B and C declare their true values, and A might modify it if she has an incentive to do so. Will she modify the value and declare it untruthfully? What value would A need to declare in order for the good not to be delivered? Would it be better for A than revealing her value truthfully?