PUBLIC GOODS

Problem 1

A small town consists of 2,000 inhabitants. The utility function of each inhabitant is $U(x_i,y) = x_i + y^{1/2}$, where x_i is the private good inhabitant *i* consumes, and *y* is the amount of the public good delivered in the town. The price of the private good is \$1 per unit, and the public good costs \$10 per unit. Each inhabitant has an income of \$2,000. What is the Pareto-efficient amount of the public good provided in the town?

Problem 2

Fifty inhabitants live in a mountain village. As a result of a fire, a playroom for children in this village has burnt down recently. The village mayor aims at building a new playroom, and he needs to decide on its size. For every village inhabitant, the marginal rate of substitution of square meters of the new playroom with money spent on other goods is $|MRS| = 1.2 - 0.0004x$, where *x* is the size of the playroom in square meters. The marginal cost of one square meter of the playroom is \$20. Find the socially efficient size of the new playroom for this village.

Problem 3

All 10 neighbors living in the same street are willing to pay 2 zł for installing each additional streetlight. The neighbors' willingness to pay for this does not depend on the number of already installed streetlights. The cost of setting up *x* streetlights is equal to $C(x) = x^2$. Find the Pareto-efficient number of streetlights installed in this street.

Problem 4

Inhabitants of a village use a common field for grazing their cows. The price of a cow is 200. The amount of milk produced by a cow depends on how many cows use the common grazing field. The milk production function is $M(C) = 300C - C^2$, where *C* is the number of cows using this grazing field and M is the liters of milk produced. The price of 1 liter of milk equals \$1 and does not depend on the amount of milk supplied.

a) What is the number of cows grazed on the common field that would maximize the joint profit of all villagers?

b) How many cows will be grazed when farmers are not able to influence others' decisions about the use of the common field, and no regulations exist regarding this use?

c) Find the difference in joint profits for the situation in b and the optimum outcome in a.

Problem 5

As a big simplification, all the goods in a certain society can be divided between private and public goods. This society consists of two groups of consumers, who differ in their demand for private goods and public goods. Functions describing their demand for these two kinds of goods have been estimated. The first group's demand function for private goods is $Q(p) = 10 - 2p$, while their demand for public goods is $X(p) = 100 - p$. The second group's demand function for private goods is $Q(p) = 8 - 0.8p$, and their demand for public goods is $X(p) = 200 - p$. In every function, *p* denotes a corresponding price (different prices for private and public goods). Show graphically and algebraically the aggregate demand functions for each of the two types of goods in this society.

Problem 6

Public safety is, as the name indicates, a public good. New residential districts are often additionally protected by guards. Two types of people live in one of such districts: younger and older ones. The first ones are generally poorer than the older ones, plus they don't feel that much need for the guards; thus, their inverse demand function for guards is given by $P(x) = 10 - 3x$, where *x* is the number of guards and *P* is the price. The inverse demand function of the older ones for guards is $P(x) = 20 - x$. The inverse supply of guards is given by $P(x) = 20 + x$.

d) Provide the formula of the function of aggregate demand for guards and present it graphically.

e) How many guards should be employed in this district?

Problem 7

In a duplex live Ann and Bob, each in their own part of the house. The city council has agreed to provide G units of a public good, which would be financed only from the inhabitants' contributions. Ann and Bob have identical utility functions: $U(X, G) = 2\ln(X) + \ln(G)$, where X denotes private consumption and G the consumption of the public good. The amount of the public good provided equals the number of units purchased by Ann and Bob together. Both Ann and Bob have an income of 200 each. The unit price of every good is 1.

a) How many units of the public good (G) will be delivered without any intervention in the market mechanism?

b) What amount of G is socially optimal?

c) Suppose the city council is unhappy with the market allocation and levied a lump-sum tax of 10 on each inhabitant. The public good is provided from the tax revenues and individual purchases as earlier. Is the Pareto efficient allocation reached then?

d) Suppose now that the city council is still unhappy with the allocation and, instead of the lump-sum tax from point c), the council charges Ann with the tax of 50 and Bob with the tax of 25. The tax revenues are again used for the provision of the public good, in addition to individual purchases. How many units of G are finally delivered? How many units of G will Ann buy, and how many will Bob buy? Compare this situation with c).